A Simple Carrier Frequency Offset Synchronization Strategy for Multiple Relay Cooperative Diversity OFDM System

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Abstract

Cooperative Diversity Orthogonal Frequency Division Modulation (CD-OFDM) systems are very sensitive to synchronization errors. In CD-OFDM, synchronization is more complex because all cooperative nodes (CNs) have their own frequency oscillator and different channel path which results in different timing and carrier frequency offset (CFO) for each node. Consequently, each node has to be synchronized separately without affecting the synchronization process of other nodes. All CNs transmit simultaneously during cooperation phase (C-phase) and their aggregate signal is received at the destination node. A unique frequency domain (FD) preamble is proposed for each CN during C-phase that will allow simple separation of cooperative nodes. These FD multiplexed preambles make the synchronization problem identical to OFDMA uplink. OFDMA system typically uses highly complex iterative CFO estimators for uplink synchronization. However, a simple one-shot CFO estimator is proposed that uses repeated preamble of two OFDM symbol duration. The proposed method is computationally efficient because it relies on FFT operation for user separation and interference mitigation. Subsequently, time domain (TD) multiplication is used for CFO correction of each CN. Furthermore, a CD-OFDM protocol for data transmission is presented that suites the proposed estimator and harnesses spatial diversity. The proposed estimator shows good statistical results during simulations in AWGN and Rayleigh environments. During evaluation, estimator variance, mean square error and symbol error rate are used as performance measure.

Key Words: Synchronization; Cooperative Diversity; OFDM; Frequency Offset Estimation; MIMO

1. Introduction

Most of the inventions in the human history have been made possible by constantly evolving and increasingly demanding life style of human race. As always, today those technologies are being developed that can support our agile and mobile life style. Telecommunications is no exception to this rule. A lot of research is being done in this field to develop technologies that can support high data rate transfer on the move.

As far as the data rate is concerned, wired communication systems using optical fiber are upto the task but falters on mobility issue. Here comes the wireless communications, which has evolved from narrowband systems to high data rate broadband systems. It is very challenging to sustain a high data rate transmission in a hostile wireless channel. Future wireless communication systems are required to provide services to mobile users for a variety of high data rate multimedia applications like VoIP, interactive gaming, video conferencing and web browsing in a wider coverage area.

The main problem with high data rate single carrier wireless systems is that as the data rate increases, the symbol duration shrinks and this results in severe inter symbol interference (ISI) specially for a dispersive fading channel. The data rate in FDMA and GSM systems is limited primarily due to this reason. For the mitigation of ISI, the symbol duration used for transmission should be increased to the extent that it exceeds the maximum delay spread (MDS) of the channel impulse response (CIR) [1].


OFDM is the technology that increases the symbol duration. An OFDM system divides the transmission channel into many narrowband sub channels that results in longer symbol duration. However, to support large data rates information bits are transmitted in parallel on these sub channels. Due to large symbol duration OFDM systems are robust against channel induced ISI. OFDM is likely to be incorporated in IEEE 802.20a standard that will provide high data rate for highly mobile users. IEEE 802.11a LAN and IEEE802.16a MAN standards are already based on OFDM technology [2].

To further increase the transmission rate, multiple transmit and receive antennas can be used to open up additional channels in spatial domain. The technology that incorporates this concept is called multiple input multiple output (MIMO) system. IEEE 802.11n WLAN and IEEE802.20 MAN standards are based on MIMO technology A MIMO system can theoretically enhance the capacity for a flat fading channel by a factor equal to the minimum number of transmit or receive antenna. Due to these reasons, the combination of OFDM and MIMO is extensively used in numerous communication systems for supporting high data rate for mobile users [3]. Theoretical studies have shown that MIMO–OFDM technology may result in highly bandwidth efficient systems to the extent of 10 bits/Hz.

The MIMO–OFDM combination has a major limitation that in most of the cases it is not feasible to use multiple antennas at the user end due to power, cost and size constraints. The solution to this problem is the use of distributed antenna nodes that cooperate with each other to form a virtual antenna array in place of physical antennas and provide spatial diversity. Spatial diversity can be effectively used to mitigate fading in a wireless channel. Such systems are known as “Cooperative Diversity (CD) Systems”. Research has shown that benefits accruing from centralized MIMO systems are also attainable by using CD system. Figure 1 shows a comparison of MIMO system and a comparable CD system. The CD systems are widely used in relay, sensor and broadcast networks [4].

The CD-OFDM system offers a number of benefits. First, OFDM makes it possible to design a simple receiver that uses low complexity frequency domain (FD) equalization to mitigate frequency selectivity by employing single multiplication per sub carrier. Second, cyclic prefix (CP) used in an OFDM symbol will make the system less sensitive to channel delay spread. Third, OFDM spreads the channel fade over multiple symbols thus making the system robust against frequency selective fading and doppler shift. Fourth, CD system using relays provides spatial diversity without using multiple antennas at the user end and results in power savings. Owing to these reasons, beyond-third generation and fourth generation systems are likely to incorporate relay protocols like amplify-and-forward (AF) and decode-and-forward (DnF). Most recently, IEEE LAN/MAN standard 802.16j has incorporated relays [5].

In order to harness all these benefits of a CD-OFDM system, all cooperating nodes need be synchronized vis-à-vis time and frequency offset. Timing asynchronism problem can be dealt with by using OFDM that is quite robust to small timing errors due to presence of CP. In OFDM, small residual timing offset (TO) results in phase shift in FD channel that is conveniently addressed by channel estimator. It is pertinent to mention that longer symbol duration in OFDM relaxes the timing synchronization problem but makes the system vulnerable to CFO. Further, inaccurate frequency synchronization results in residual frequency offset thus making the channel time variant. It results in inter carrier interference (ICI).

CFO problem for single user OFDM system has been extensively explored in literature [6] and
similarly many methods have been presented for OFDMA [7].

![Diagram of broadcast and cooperation phases](image)

**Fig.2** (a) In Broadcast phase only source node transmits. Solid and dashed link lines indicate successful and failed decoding respectively at a nodes. Nodes are shaded to indicate the successful receipt of NAK signal from destination. (b) In Cooperation phase, only those nodes participate that have successfully decoded the source transmission and have also received the NAK from destination.

CFO estimation for CD-OFDM system is still open for research and is drawing attention of researchers. The estimation of CFO in cooperative environment turns out to be a multiple parameter estimation problem and its maximum likelihood (ML) solution requires a multidimensional (M-D) space search. It makes the exact ML solution extremely complex and researchers resorted to transforming M-D search problem into a series of single dimension searches [8], [9] and [10]. These iterative CFO estimation algorithms are still computationally complex and require proper initialization for convergence. Choi [11] proposed a reduced complexity, post FFT scheme for CFO correction in multiple-user environment that used circular convolution. However, this scheme cannot be used for timing correction and suffers due to multiple access interference (MAI). The MAI issue in Choi algorithm was addressed by incorporating an iterative interference cancellation scheme [12] but complexity was substantially increased. Some non-iterative methods were also proposed for CFO estimation and mitigation. One such CFO mitigation algorithm used long CP mitigation but required complex matrix inversion for each FFT block [13]. Similarly, an algorithm [14] used virtual subcarriers and subspace decomposition CFO estimation. Another method is proposed in [15] that pre-corrects respective CFOs at relays instead of destination but this algorithm is based on assumption that relay knows CFO between it’s receive and transmit paths.

In this paper, a CFO estimator and corrector is proposed for CD-OFDM system. A novel idea similar to OFDMA uplink frequency offset estimation/correction is employed in cooperative scenario. We have presented a simple one shot CFO estimator for cooperative scenario that uses FD preamble of two repeated OFDM symbols.

![Frame structure of CD-OFDM](image)

**Fig.3** Frame Structure of CD-OFDM comprising broadcast and cooperative subframes. FD preamble for source and M relay nodes are also shown.

Paper organization is as follows. Section 2 introduces the CD-OFDM system for DnF mode. CFO estimation and correction are developed in section 3. Section 4 presents the simulation results to validate the proposed algorithms. Finally, paper is concluded in section 5.

### 1.1 Notations

Small letters are used for time domain (TD) signals and capital letters are used for frequency domain (FD) signals. Real and imaginary components are indicated by $\text{Re} \{ \}$ and $\text{Im} \{ \}$ respectively. $E\{ \}$, $\text{conj} \{ \}$ and $\text{IFFT}\{ \}$ denotes expectation, conjugate and inverse fast Fourier transform operations respectively. DnF is used to represent Decode and Forward mode.
2. System Description

2.1 Cooperative Diversity Protocol

We use two-phase cooperative system comprising: broadcast phase (B-phase) and cooperation phase (C-phase) as described in [16], [17]. The system comprises of a destination node (DN), a source node (SN) and M relay nodes (RNs) as shown in Figure 2. All RNs operate in DnF mode.

In the B-phase, only SN broadcasts preamble followed by information blocks using all OFDM subcarriers. The RNs and DN attempt to decode it and failure or success in decoding is determined by computing the frame check sum (FCS). The DN transmits acknowledgment (ACK) or no-acknowledgement (NAK) signal based on result of decoding. The ACK/NAK signal is used for RN selection in next phase and for coarse timing and frequency synchronization at SN and RNs. This arrangement ensures synchronous arrival of data at DN and synchronous detection.

During the C-phase, if an ACK is transmitted by DN then SN and RNs do not transmit. However if DN transmits a NAK then M RNs that have successfully decoded the SN transmission and NAK from DN start transmission simultaneously. Each RN transmits its unique FD preamble and data block on CAS mapped subcarriers. Any number of RNs can participate in C-phase but we have used two RNs (i.e. \( M = 2 \)).

During C-phase, DN receives a signal that is aggregate of all signals transmitted from RNs. Consequently, DN faces multiple parameter estimation problem because each RN operates independently with its separate oscillator and channel path. It results in independent CFO and TO for each RN. It makes the synchronization problem in C-phase identical to that of uplink OFDMA.

B-phase synchronization is similar to single user OFDM synchronization and any of the known method can be used for synchronization [6]. In this paper, we have only tackled the C-phase CFO synchronization problem being more challenging.

2.2 Frame Structure and Preamble

Transmission of a single frame is completed in two phases. Transmitted frame comprises of two subframes: broadcast subframe and cooperation subframe [16] that are transmitted using OFDM. Frame structure and preamble are shown in figure 3. Each subframe comprises of data and a FD broadcast preamble (FDBP) or cooperation preamble (FDCP) that is used for CFO estimation.

In B-phase, FDBP is composed of \( N_{pt} \) repeated FD training symbols \( \{ P_s[n]: 0 \leq n \leq N_u - 1 \} \) from complementary golay (CG) sequence . The index set of subcarriers assigned to \( P_s \) is \( \mathcal{F}_s \subset \{-N_u, -N_u + 1, \ldots, \frac{N_u}{2} \} \). The cardinality of this set is \( |\mathcal{F}_s| = N_u \). CAS mapped data block follows the FDBP.

In C-phase, index set of subcarriers assigned for \( q_{th} \) RN is \( \mathcal{F}_{R,q} \) and depends on CAS. The FDCP of \( q_{th} \) node is unique and is formed by placing null on all subcarriers except those included in \( \mathcal{F}_{R,q} \). Assuming \( M \) active nodes, the cardinality of index set for \( q_{th} \) RN is \( |\mathcal{F}_{R,q}| = N_u/M \). Note that only those values of \( M \) are allowed that result in integer values for \( |\mathcal{F}_{R,q}| \). In case of sub-band CAS with \( = 2 \), index set for first relay is \( \mathcal{F}_{R,1} = \{-N_u, -N_u + 1, \ldots, -1\} \) and index set for second relay is \( \mathcal{F}_{R,2} = \{1, 2, \ldots, \frac{N_u}{2}\} \). The FD training symbol of a specific RN (i.e. \( P_{R,s} \)) is formed by placing null at all subcarriers excluded from respective subset and placing respective CG sequence value on remaining active subcarriers.

\[
P_{R,x}(l) = \begin{cases} 
P_s(l), & l \in \mathcal{F}_{R,x} \\
0, & \text{else} 
\end{cases}
\]
Where \( \frac{N_u}{2} \leq l \leq \frac{N_u}{2} \) 

(2)

Then IFFT operation is performed on this FD training symbol to get respective TD OFDM training symbol. Then \( N_{pd} \) periods of this TD OFDM training symbol constitute the complete TD preamble that is followed by data block. Each RN transmits its data on its pre-assigned subcarriers. Figure 4 shows FD preamble for two and four RN cooperating system.

The received signal during C-phase is a superposition of transmitted signals from all RNs. The unique preamble for each RN will allow us to separate the cooperating nodes at the destination end for CFO estimation and correction.

![Block diagram of CD-OFDM transmitter and receiver](Image)

\( \text{Fig.5} \) Block diagram of CD-OFDM transmitter and receiver. Dotted blocks are used only during cooperative phase. Shaded blocks show the CFO estimation and correction modules.

### 2.3 CD-OFDM signal and Channel Model

A simplified block diagram for transmitter and receiver is shown in figure 5 that only shows system components necessary for explaining the proposed algorithm. In our model, we will consider the C-phase, where multiple RNs are transmitting simultaneously and this aggregate signal is received at DN.

All nodes are based on OFDM system using \( N \) point inverse fast Fourier transform (IFFT). Raw data samples are modulated using phase shift keying modulation to form FD symbols \( a_{n,k} \ldots a_{n,k} \) is the symbol mounted on \( k^{th} \) subcarrier frequency and \( n^{th} \) OFDM symbol. Then \( N_u \) \( (N_u \leq N) \) of these symbols are fed in parallel to an IFFT module to form OFDM symbol IFFT is implemented as a discrete operation so output is discrete time samples with sampling period \( T = \frac{T_s}{N} \). The subcarriers at both ends of the OFDM symbol are left unused as guard bands. Subsequently, transmitted signal is band limited to less than \( \frac{1}{T} \) bandwidth by using a transmit filter \( G_T(w) \) thus making it simpler to design a T-spaced receiver [2].

There are \( M \) RNs and each node transmits its signal independently on CAS mapped subcarriers. \( a_{n,k} \) is null for non-assigned subcarriers. The complex baseband signal transmitted by \( x^{th} \) RN is described as:

\[
s_x(t) = \frac{1}{\sqrt{T_u}} \sum_{k=\frac{N_u}{2}}^{\frac{N_u-1}{2}} a_{n,k} * \psi_{n,k}(t) * g_T(\tau)
\]

(3)

where \( x = \{1, 2, \ldots, M\} \).

The pulse shaping of all subcarrier symbols in an OFDM symbol is carried out by using a rectangular pulse of length \( T_u \) and are modulated on a baseband subcarrier with frequency \( f_k = \frac{k}{T_u} \). The carrier spacing is \( f_\Delta = f_k - f_{k-1} \). A cyclic prefix (CP) of duration \( T_g = N_g T \) is added to signal to reduce ISI. Hence, complete OFDM symbol length becomes \( T_c = N_u + T_g \). Note that \( N_g \) is the number of CP samples and \( T_g \) is the duration of CP. Now the subcarrier pulses are:

\[
\psi_{n,k}(t) = e^{j2\pi(\frac{1}{N})(t-T_g-n_{T_g})}
\]

(4)

\[
u(t) = \begin{cases} 1 & 0 \leq t \leq T_c \\ 0 & \text{else} \end{cases}
\]

(5)

Finally, the equivalent representation for the \( k \) samples of transmitted baseband \( n_{th} \) OFDM symbol \( s_x(k) \) is as (3) and OFDM symbol length in a sample spaced system is \( N_c = N_u + N_g \).

\[
s_x(k) = \frac{1}{\sqrt{N}} \sum_{l=\frac{-N_u}{2}}^{\frac{-N_u-1}{2}} a_{n,k} * e^{j2\pi(\frac{l}{N})}
\]

(6)

\(-N_g \leq k \leq N - 1\)
Consider a frequency selective multipath fading channel \( h_x(\tau, t) \) that combines the effect of actual channel impulse response (CIR) and transmit filter \( g_T(\tau) \).

\[
h_x(\tau, t) = \sum_{i} h_{x,i}(t)^* \delta(\tau - \tau_i) \quad (7)
\]

\( i = 0, 1, \ldots, K_x - 1 \)

Note that \( h_{x,i}(t) \) and \( \tau_i \) are the complex path gain and delay at time \( t \) for link from \( x^{th} \) RN to DN. The maximum channel delay spread (MCDS) is \( \tau_{\text{max},x} = \tau_{\text{i}=K_x-1} \) and \( K_x \) is the length of channel impulse response for link from \( x^{th} \) RN to DN. For sample spaced channel, \( \tau_i = i \) and \( h_j \) represents path delays and discrete-time channel impulse response respectively. Assuming a flat receive filter, the receiver input signal at DN \( (D) \) from single \( x^{th} \) RN is:

\[
r_{D,x}(t) = \sum_{i} h_{x,i}(t)^* S_x(t - \tau_i) + w(t) \quad (8)
\]

The equivalent representation for the \( k \) samples of received baseband \( n_{\text{ch}} \) OFDM symbol is:

\[
r_{D,x}(k) = \sum_{i=0}^{K_x-1} h_{x,i}^* S_x(k - \tau_i) + w(k) \quad (9)
\]

where \( w(k) \) is complex Gaussian distributed noise process. Noise samples are assumed to have zero mean and \( \sigma_w^2 \) variance.

The geometric gain \( G_{x,y} \) for \( x^{th} \) RN to SN or DN is as under [17]. \( SD \) indicates link from SN to DN.

\[
G_{x,y}(k) = \frac{E\{\sum_{i=0}^{K_x-1} |h_{x,i}|^2\}}{E\{\sum_{i=D}^{K_S-1} |h_{SD}[i]|^2\}} \quad (10)
\]

3. Proposed Carrier Frequency Synchronization Method

The CFO synchronization in a cooperative scenario is different from a single user case. First, all RNs are spatially separated and have separate oscillators with independent oscillator drifts. Second, all RNs have an independent channel path with different path delays. Third, all RNs transmit asynchronously unless there is a mechanism to ensure synchronous transmission. This problem is further exaggerated due to different path lengths for each RN to DN link. Fourth, all RNs transmit simultaneously by sharing time and frequency domain so user separation at destination is quite challenging. User separation is required for CFO estimation of each RN. Finally, even if somehow CFO estimation for each RN is successfully carried out then CFO correction at DN for one RN will introduce CFO in other RN signals. The proposed CFO estimation and correction method will take care of all these issues.

In order to address the asynchronism problem, TO and CFO information from DN is fed back to SN and RNs for local synchronization. It eases the asynchronism and synchronization problem a bit. To further relieve the timing synchronization at DN, the CP length is extended to the extent that it accommodates the combined effect of the MCDS and maximum two-way propagation delay \( \rho_{\text{max}} = \max(\rho_x : x = 1, 2, \ldots, M) \). It is accomplished by making \( N_R \geq \tau_{\text{max}} + \rho_{\text{max}} \). Also, note that \( \tau_{\text{max}} = \max(\tau_{\text{max},x} : x = 1, 2, \ldots, M) \). Such a system is known as quasi-synchronous and timing errors can be handled as part of channel response.

3.1 Carrier Frequency Offset Estimation

At the DN, receiving antenna superimposes all RN signals to produce an aggregate signal that is represented in baseband as:

\[
r_D(k) = \sum_{x=1}^{M} r_{D,x}(k) + w(K) \quad (11)
\]

Taking symbol timing offset and carrier frequency offset into consideration, the received signal \( r_{D,x}(k) \) for C-phase from \( x^{th} \) RN becomes:

\[
r_{D,x}(k) = \exp(j \varphi_x) \exp(j \frac{2\pi k v_x}{N_S}) \sum_{i=0}^{K_x-1} h_{x,i}^* 
\]

\[
S_x(k - \tau_{x,i} - \theta_x) \quad (12)
\]

where \( v_x \) is the carrier frequency normalized by the \( f_A \), \( \theta_x \) is timing error with integer values, \( \varphi_x \) is an arbitrary carrier phase factor, \( h_{x,i} \) is a specific channel impulse response tap. \( \tau_{x,i} \) is timing offset for
th x RN. Note that $\phi_k$ cannot be distinguished from phase shift introduced by channel and therefore is assumed to be absorbed in channel effect and subsequently compensated by channel estimation module.

We are only dealing with CFO synchronization problem so it is assumed that system is already time synchronized.

$$r_{D,x}(k) = \exp\left(j \frac{2\pi k_v x}{N} \right) \sum_{i=0}^{K_s-1} h_{x,i} s_x(k) \ast (k - \tau_{x,i}) \quad (13)$$

Note that noise contribution from each RN is already incorporated in $r_{D,x}(k)$. The CFO estimation is based on a $N_{pd}$ times repeated FD preamble that is unique for each RN.

It is assumed that channel remains constant for $N_{pd}$ OFDM symbol duration. For sake of brevity, $N_{pd} = 2$ is used but can be extended to any length by incorporating minor changes in derivations that follow. Increasing $N_{pd}$ will increase the overhead but will improve the estimation because of averaging effect. Due to extended CP, the received samples placed in the DFT window are free from ICI and two repeated training symbols $r^{(0)}_{D,x}(k)$ and $r^{(1)}_{D,x}(k)$ received for $x^{th}$ RN are:

$$r^{(0)}_{D,x}(k) = \exp\left(j \frac{2\pi k_v x}{N} \right) \sum_{i=0}^{K_s-1} h_{x,i} s_x(k) \quad (14)$$

$$r^{(1)}_{D,x}(k) = \exp\left(j \frac{2\pi k_v x}{N} + j \frac{2\pi N c}{N} \right) \sum_{i=0}^{K_s-1} h_{x,i} s_x(k) \quad (15)$$

After FFT, for AWGN channel, aggregate received repeated symbols $R^{(0)}_{D}(l)$ and $R^{(1)}_{D}(l)$ are:

$$R^{(0)}_{D}(l) = P^{(0)}_{S}(l) \quad (16)$$

$$R^{(1)}_{D}(l) = P^{(1)}_{S}(l) \ast \exp\left(j \frac{2\pi N c}{N} \right) \quad (17)$$

Note that after FFT, the subcarrier index $l$ is used instead of $k$. Now FD training subcarriers for each RN can be easily separated as their respective subcarrier index sets are known. $P^{(0)}_{S}(l)$ and $P^{(1)}_{S}(l)$ are received repeated composite preamble symbols with added noise and in case of noise free channel both are equal.

$$R^{(0)}_{D,x}(l) = P^{(0)}_{R,x}(l) + \text{noise} \quad (18)$$

$$R^{(1)}_{D,x}(l) = P^{(1)}_{R,x}(l) \ast \exp\left(j \frac{2\pi N c}{N} \right) + \text{noise} \quad (19)$$

Now first and second received training symbol of a specific RN (i.e. $P^{(0)}_{R,x}(l)$ and $P^{(1)}_{R,x}(l)$) are separated from the composite preamble as:

$$P^{(0)}_{R,x}(l) = \begin{cases} P^{(0)}_{S}(l), & l \in P_{R,x} \\ 0, & \text{else} \end{cases} \quad (20)$$

$$P^{(1)}_{R,x}(l) = \begin{cases} P^{(1)}_{S}(l), & l \in P_{R,x} \\ 0, & \text{else} \end{cases} \quad (21)$$

Where

$$-\frac{N_x}{2} \leq l \leq \frac{N_x}{2} \quad (22)$$

All null values are excluded and $\frac{N_x}{M}$ subcarrier values from each repeated training symbol (i.e. $P^{(0)}_{R,x}(l)$ and $P^{(1)}_{R,x}(l)$) are used for CFO estimation.

For a specific RN, it is evident that two repeated portions are affected in a same way by channel and the only difference between their respective values is $\frac{2\pi N c}{N}$. This difference expression is further simplified when $N_c = N$. This fact makes it possible to estimate the CFO of each RN in cooperative scenario in a way similar to single user estimation [6]. The estimated CFO $\tilde{\nu}_x$ for each RN can be calculated as:

$$\tilde{\nu}_x = \frac{1}{2\pi N_x} \tan^{-1}\left\{ \frac{\sum_{i=1}^{N_x} \text{Im} \{Q_{x,i} \}}{\sum_{i=1}^{N_x} \text{Re} \{Q_{x,i} \}} \right\} \quad (23)$$

$$Q_{x,i} = \tilde{P}^{(1)}_{R,x}(i) \ast \text{conj} \{ \tilde{P}^{(0)}_{R,x}(i) \} \quad (24)$$

Where

$$1 \leq i \leq \frac{N_x}{M} \quad (25)$$
3.2 Carrier Frequency Offset Correction

CFO estimate for each RN is obtained by using the proposed method but CFO correction cannot be directly applied to the aggregate signal. Let \( R_{R_X}^{(n)} \) be the \( n \) received information symbol in FD that is formed by placing null at the subcarriers not included in index set of \( x \) RN. Placement of nulls on non-member subcarriers results in noise suppression because originally no information was transmitted on those subcarriers by \( x \) RN.

\[
R_{R,X}^{(n)}(l) = \begin{cases} 
C_{R_X}^{(n)}(l), & l \in P_{R,X} \\
0, & \text{else}
\end{cases} \quad (26)
\]

Where\( \quad 2 \leq n \leq \text{data block size} \quad (27) \)

Now take IFFT of \( I_{R_X}^{(n)} \) and then apply CFO correction using \( \hat{\nu}_x \) to this TD signal \( \hat{I}_{D_X}^{(n)} \) as:

\[
\hat{I}_{D_X}^{(n)} = \text{IFFT} \left( P_{R_X}^{(n)}(l) \right) \exp \left( -j \frac{2\pi}{M} \right) \quad (28)
\]

Same procedure is repeated in parallel for all RNs.

4. Performance Evaluation, Simulation Results and Discussion

The performance of proposed algorithms is evaluated through computer simulations and it is assumed that exact number of participating relays is known to destination. The FD preamble is composed of two (i.e. \( N_{pd} = 2 \)) repeated OFDM symbols. CG sequence of length \( N \) is used for forming training symbol. The proposed preamble exhibits good peak to average power ratio (PAPR).

\[
0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5
\]

\[
0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60
\]

\[
10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1}
\]

\[
0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60
\]

\[
10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 10^{-1} \quad 10^0
\]

\[
0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5
\]

\[
0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60
\]

\[
10^{-6} \quad 10^{-4} \quad 10^{-2} \quad 10^{-1} \quad 10^0
\]

Fig.6 Average CFO estimate using proposed estimator versus actual CFO.

The cooperative diversity network comprises of one SN, two RNs (i.e. \( M = 2 \)) and one DN. Geometric gain \( G_l \) of 0dB is used for all considered cases for fair comparison. All nodes are OFDM based systems with 1024 subcarrier and 10% guard interval. QPSK is used for baseband modulation. All simulations are run at least 10,000 times in baseband and no pulse shaping or frequency up-conversion is done. The normalized CFO is restricted to less than half of the carrier spacing. The AWGN and Rayleigh channel with three taps is used for simulation. For performance comparison (figure 11) with other well-known algorithms, some of the parameters were modified for tractable and fair comparison.

In figure 6, average of estimated CFO \( E[\hat{\nu}_x] \) calculated by using the proposed estimator is plotted against actual relative CFO for 20dB SNR in AWGN channel. Plot shows that proposed estimator accurately estimates the actual CFO and there is no ambiguity in half carrier spacing range. It is pertinent to mention that support for CFO within half carrier spacing makes the algorithm robust. Whenever, channel is benign enough to have small CFO then system parameters can be changed to have dense sub-carriers thus affording higher throughput.

Fig.7 Standard deviation of estimated CFO versus SNR

Fig.8 Variance of estimated CFO versus actual CFO for single user and two cooperating relay nodes in AWGN environment (20 dB).
In figure 7, standard deviation (STD) of estimated CFO $\hat{\nu}$ is plotted versus SNR for AWGN and multipath channel. STD hits a floor in case of multipath due to dispersive nature of the channel. Note that proposed estimator was actually derived for AWGN but has shown comparable performance for multipath in practical SNR range.

In figure 8, variance of estimated CFO is plotted against different values of actual CFO. Results show that variance remains constant for different CFO values for single user and two-user case. Variance only shoots up when we approach the half carrier spacing limit.

Figure 9 shows the effect of increasing number of RNs on estimator variance. Remarkably, variance almost remains constant when number of RNs in a cooperating environment is increased from one to eight.

In figure 11, the performance of proposed algorithm (i.e. “New”) is compared with “HL” [12], “LFH” [13] and “CLJL” [11] in terms of symbol error rate (SER). The FFT size of 64 is used and accurate time synchronization is assumed where applicable for concentrating on performance of CFO algorithms. All algorithms were modified to exclude convolutional coding and incorporate Alamouti STBC. For fair comparison, the HL algorithm was constrained to single iteration only. Reason being that HL algorithm takes its initial estimate from CLJL algorithm for iteration so increasing number of iterations may favour it out of proportion. Moreover, even single iteration HL is computationally more costly as compared to other algorithms. However, for sake of completeness, HL algorithm was run for four iterations (result not shown in Figure 11) and significant observation was an improvement in performance for HL(0.4) and it converged towards HL(0.1). Increase in iterations resulted in slight improvement for HL(0.1). Moreover, different bandwidth efficiency of these algorithms was compensated by altering the transmission power. Figure 11 shows that proposed algorithm “New(0.1)” performs better than other algorithms in the practical SNR values (i.e. 12~18 dB) for average normalized CFO value of 0.1. Even for the case of higher CFO value of 0.4, the “New(0.4)” algorithm performs better than “HL” and “CLJL”, specially at large SNR values. The performance of “LFH” is understandably better for higher SNR and especially for high CFO values owing to larger CP and computational complexity. However, it is included in comparison for the purpose of completeness.

Figure 10 shows the effect of change in total number of cooperating RNs on mean squared error (MSE) of proposed estimator. Mean squared error increases as the number of RNs is increased from one to twenty RNs but thereon becomes steady. Similarly, MSE increases with decrease in SNR value.

Figure 11 shows symbol error rate versus SNR for different carrier frequency offset estimators in cooperative environment. The proposed algorithm (New) performs better in practical SNR range. Solid and dashed lines are used to differentiate between 0.1 and 0.4 CFO cases.
5. Conclusion

The OFDM and MIMO technologies have been successfully used in a number of wireless standards to provide high data rate in a multipath fading environment. However, in mobile communications, it is not feasible to have multiple antennas at the user end. In such scenario, cooperative diversity systems may be considered as an attractive alternative. CD-OFDM systems may be widely adopted in future mobile communication systems due to their favorable features. In a step towards that direction, this paper presented and evaluated performance of a FD preamble that especially suits CD-OFDM systems using DnF relay protocol. Moreover, CFO estimation algorithm and CFO correction method are also described and performance evaluated. Simulations verify that proposed estimation algorithm shows good results in terms of mean square error, variance of estimator and symbol error rate. Proposed algorithm effectively uses separate training for each relay and is bandwidth efficient as compared to [13] and much less computationally intensive as compared to [10]-[15].

6. References


[13] X. Li, F. Ng and T. Han, "Carrier frequency offset mitigation in asynchronous cooperative OFDM transmissions," IEEE Transactions on

